Mixed Strategies and Nash Equilibrium

Game Theory Course:
Jackson, Leyton-Brown & Shoham

Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies

	Heads	Tails
Heads	$1, -1$	$-1, 1$
Tails	$-1, 1$	$1, -1$

Mixed Strategies

- It would be a pretty bad idea to play any deterministic strategy in matching pennies
- Idea: confuse the opponent by playing randomly
- Define a strategy s_i for agent i as any probability distribution over the actions A_i.
  - pure strategy: only one action is played with positive probability
  - mixed strategy: more than one action is played with positive probability
    - these actions are called the support of the mixed strategy
- Let the set of all strategies for i be S_i
- Let the set of all strategy profiles be $S = S_1 \times \ldots \times S_n$.

Utility under Mixed Strategies

- What is your payoff if all the players follow mixed strategy profile $s \in S$?
  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell

Utility under Mixed Strategies

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  - We can't just read this number from the game matrix anymore: we won't always end up in the same cell
- Instead, use the idea of expected utility from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) Pr(a|s)$$

$$Pr(a|s) = \prod_{j \in N} s_j(a_j)$$

Best Response and Nash Equilibrium

Our definitions of best response and Nash equilibrium generalize from actions to strategies.

Definition (Best response)

$s_i^* \in BR(s_{-i})$ iff $\forall s_i \in S_i,\ u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i})$

Definition (Nash equilibrium)

$s = \langle s_1, \ldots, s_n \rangle$ is a Nash equilibrium iff $\forall i,\ s_i \in BR(s_{-i})$

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Theorem (Nash, 1950)

Every finite game has a Nash equilibrium.

Example: Matching Pennies

	Heads	Tails
Heads	$1, -1$	$-1, 1$
Tails	$-1, 1$	$1, -1$

Example: Coordination

	Left	Right
Left	1, 1	0, 0
Right	0, 0	1, 1

Example: Prisoner's Dilemma

	C	D
C	$-1, -1$	$-4, 0$
D	$0, -4$	$-3, -3$